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Aligned fields, magneto-fluid dynamic drag measurements with large interaction parameter

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The drag of spheres and disks has been measured in a flow of liquid sodium within an aligned magnetic field. A slightly viscous, strong field limit is discussed and explored experimentally. In this limit the drag coefficient was found to have an asymptotic dependence proportional to the square root of the interaction parameter, (the ratio of magnetic to inertial force, independent of body shape). A physical model is presented along with preliminary verification of its basic characteristics.

Author

1. Introduction

Although the problem of aligned fields magneto-fluid dynamic (MFD) flow past bodies has received much attention from applied mathematicians in the last ten years, there has been a general lack of emphasis on physically realisable cases. This situation has been partly due to analytical difficulties and partly due to the limited interest and success of the experimenters themselves. The steady dimensionless equations for a viscous incompressible conducting flow are

$$\underline{V}\cdot\nabla\underline{V} + \nabla\underline{P} = N(\underline{V}\times\underline{B})\times\underline{B} + Re^{-1}\nabla^2\underline{V}$$

$$\nabla\times\underline{B} = Rm(\underline{V}\times\underline{B})$$

$$\nabla\cdot\underline{B} = \nabla\cdot\underline{V} = 0$$

where $\underline{V} = \underline{V}'/V_o$, $P = (P' - P_o)/V_o^2\rho$, $x = x'/d$, $B = B'/B_o$, and $N = B_o^2 \sigma d/\rho V_o$, the interaction parameter; $Re = V_o\, d/\nu$, the Reynolds number; and $Rm = \sigma \mu V_o d$, the magnetic Reynolds number. Although these nonlinear coupled equations have not been solved for arbitrary values of the three parameters, various limiting cases have been considered in great detail. To date such problems have shed little light on physically relevant problems but have in many cases served to direct the attention from them.

It is important to consider the coupling of the equations under arbitrary conditions; Van Blerkom (1960) and Gourdine (1961) have studied this by considering an Oseen-type approximation. This yields two rotational velocity perturbation modes $U_{1,2}$ satisfying $(\nabla^2 - \lambda_{1,2})\, U_{1,2} = 0$, where $\lambda_{1,2}$, two effective Reynolds numbers, are given by $\frac{Re}{2}\left[1 + \frac{Rm}{Re} \pm \left\{(1 + \frac{Rm}{Re})^2 - 4\,\frac{Rm}{Re}\,(1 - \frac{N}{Rm})\right\}^{\frac{1}{2}}\right]$.

Table 1 shows the approximate values of the parameters which appear in $\lambda_{1,2}$ under reasonable laboratory conditions.

Table 1

$V_o = .50$ m/sec , $B_o = 1$ web/m^2 , $d = .01$m

	Na (135°C)	NaK (20°C)	Hg (20°C)
Rm	.07	.02	.006
Re	8,000	6,000	40,000
N	200	70	2.0

One finds that Rm/Re , a material property, is always much less than one, and that N/Re , typically quite small, can be made ~ 1 in an actual experiment only with great difficulty. The one ratio that can be varied over a wide range is $N/Rm = B_o^2/\rho\mu V_\infty^2$ the square of the ratio of the Alfvén speed to the free stream speed. For $N/Rm \ll 1$, super-Alfvénic, $\lambda_{1,2}$ = Rm, Re, both modes give downstream wakes, and since $Rm < 1$ in typical experiments, the dominant mode is the ordinary viscous wake. We expect that in such a case the flow would only be slightly modified, if at all, by MFD effects, and is of little interest.

On the other hand, for $N/Rm \gg 1$, sub-Alfvénic, strong effects do occur and fall into two classes depending on the value of N/Re. For $N/Re \gg 1$, a clearly non-physical limit, $\lambda_{1,2} = -(NRe)^{\frac{1}{2}}, +(NRe)^{\frac{1}{2}}$, symmetric magneto-viscous wakes exist, and Chester (1961), Chang (1963), and Childress (1963) have considered the problem in detail. The other class of sub-alfvenic flows, $N/Re \ll 1$ is physically meaningful and yields $\lambda_{1,2} = -N, +Re$. If $N \ll 1$ the forward wake is very diffuse, the perturbations from potential flow upstream are small, and regular perturbation methods have been successful in treating such problems (Reitz and Foldy (1961)). On the other hand, for $N \gg 1$, large perturbations exist within the concentrated forward wake. The present work is concerned principally with this case for which neither a satisfactory theoretical treatment, nor previous experiments exist.

If we further consider the limit $N \to \infty$, $Re \to \infty$, the Oseen analysis yields simply $\partial/\partial x(U_{1,2})=0$. The statement then that perturbation quantities are independent of the coordinate along the applied field is analogous to that of the Taylor-Proudman theorem which applies to the case of strongly rotating, slightly viscous flow. The analogy has in the past been limited to the case of infinite conductivity, Chandrasekhar (1961), but the above limit for finite conductivity and strong field yields the same conclusion. Therefore one would expect stagnant slugs similar to the "Taylor Column" to form.

Liepmann, Hoult, and Ahlstrom (1960) described unsuccessful attempts to measure the drag of freely rising spheres in a tank of mercury within a solenoid. Ahlstrom (1962), using the same facility, demonstrated the existence of the forward wake, but found that wall effects were dominant for $N \leq 1$. Maxworthy (1962) measured the drag of spheres falling within a similar facility using liquid sodium, finding a marked increase in drag for strong fields. His work, however, can only be considered qualitatively due to various experimental difficulties he encountered (private communication). Motz (1965) measured the drag of a nonconducting sphere undergoing small amplitude oscillations in a container of mercury within a solenoid for the case $N < 1$ and found good agreement with a linear inviscid analysis.

The goal of this experiment was to measure the drag of bodies under a wider range of N than has been previously considered. To do this a strain gauge drag balance and wire suspension system were used in the Jet Propulsion Laboratory sodium flow facility.

2. Experiment

The sodium flow facility, which is similar to an ordinary water tunnel in its basic characteristics, has been described in detail by Maxworthy (1961). Liquid sodium is circulated in a closed loop which is maintained at a temperature of 135°C. and the flow rate is measured with a conventional crossed field flow meter. The test section, (Fig. 1), is surrounded by an oil-cooled solenoid which provides a field uniform to within 3% over 70% of its length. An entrance nozzle is employed to minimize the currents and vorticity which are induced as the fluid enters the fringe field. The flow field within the test section has been studied for various flow and field conditions using a conventional pitot tube technique. Maxworthy (1967) discusses the suitability and limitations of the use of this facility for studies of flows past bodies.

He concludes that the flow field is sufficiently uniform to permit measurements within the range of conditions employed here, 90 cm/sec $< V_o <$ 1300 cm/sec and $B_o < .70$ web/m^2 .

The test bodies were suspended from three tungsten rhenium wires which pass through 1/16" holes in the test section wall into a chamber where they are attached to three aluminum beams. A temperature compensated strain gauge bridge composed of matched encapsulated foil gauges is attached to one of the beams. The chamber is packed with silicone grease to damp beam oscillations and to protect the gauges. The balance is dead weight calibrated before and after each run at the operating temperature which is monitored by a thermocouple attached to one beam.

The material properties for sodium at the operating temperature were found by interpolation from data appearing in the Liquid Metals Handbook (1955). They are as follows: $\rho = .918 \times 10^3$ Kg/m^3 , $\sigma = .911 \times 10^7$ mho/m , $\rho\nu = .575 \times 10^{-3}$ Kg/m sec . This gave the following ranges of the three parameters: $0 < N < 80$, $10^4 < Re < 25 \times 10^4$, $.10 < Rm < 2.50$. The following configurations were utilized: .500-in. sphere, .500-in. disk, and .750-in. sphere with .010-in. suspension wires, and .250-in. sphere, .500-in. sphere with .005-in. wires.

In calculating the body drag coefficient we must subtract the wire drag from the total force measured. We assumed that the wire drag coefficient was not affected by the field and used values for this quantity given in Schlichting (1960). This was based on the assumption that the separated flow past the wires would not be measurably affected for N , based on wired diameter, $\lesssim 1$. This condition was maintained throughout, and the assumption was supported by the measurements themselves.

The drag balance, although designed as a steady device, also served to indicate the unsteadiness of the wake. The unsteady component of the force was amplified and recorded on an oscillograph but no attempt was made to separate the unsteady drag and lift, or to quantify this measurement.

3. Results and Discussion

Figure 2 shows the drag coefficient, C_D, of the spheres plotted as a function of the interaction parameter, N. It appears that, for a given d/D, where D is tunnel diameter, C_D is only a function of N and that weak and strong field regimes exist. For $0 < N < 1$, $N/Rm \lesssim 1$, the C_D remains relatively unchanged, and the points at the left of each graph indicate the scatter in the measured value of C_D for $B = 0$. The no-field C_D increases with d/D, due to blockage constraint, and the extrapolated value to $d/D = 0$, $C_D = .40$, agrees with the experimental data of previous investigators for the range $10^4 < Re < 25 \times 10^4$. One can conclude that ordinary separated flow dominates the small N regime and that magnetic forces have a negligible effect.

We must point out that this weak field regime is not well defined in terms of the conditions given earlier since both N and N/Rm are of the order 1 or smaller. It is desirable to investigate the behavior of C_D for N/Rm large as N alone varies through 1, and this has been done by Motz who found that C_D was 10% lower than the potential flow prediction for $N \backsim .10$.

The intermediate range, $1 < N < 10$, $N/Rm \gtrsim 1$, showed no simple power law dependence on N and is likely to be characterized by both MFD and ordinary separated flow effects being important. Although this range may contain several interesting aspects, it is doubtful that any simple physical picture will be suitable in understanding them.

The strong field regime, $N/Rm >> 1$, $N > 10$, is well defined and gave C_D proportional to $(N)^{\frac{1}{2}}$. We again noted the effect of blockage constraint and we may assume that it has the simple effect of increasing the effective velocity which should be used in calculating the appropriate nondimensional parameters. This would then predict that $C_D \propto (1 - \frac{d^2}{D^2})^{3/2} (N)^{\frac{1}{2}}$. The results of such a correction is shown in Fig. 3. An extrapolation to $d/D = 0$ gave $C_D = .33\,(N)^{\frac{1}{2}}$ and this is indicated by the solid line in the figure. The suitability of a simple solid body blockage correction strongly implies that the wake spreads so slowly so as not to interact with the walls and that end effects are not important. The latter, due to the finite length of the test section, would presumably have an increasingly important effect as N is further increased.

In order to investigate the effect of body shape, the drag of a 0.500-in. disk with .010-in. suspension wires was measured, and the results are shown in Fig. 4 together with that for the 0.500-in. sphere: The no-field case shows a considerable blockage, but agrees well with a partly empirical correction given by Maskell (1963), which includes the effect of the wake interacting with the tunnel wall. Again there is little effect for $N < 1$, but C_D does in fact decrease slightly for $1 \leq N \leq 10$. The field in this intermediate range seems to have the effect of reducing the wake blockage without appreciably affecting the overall drag. For $N > 10$, the disk C_D is very close to that of the sphere and the same asymptotic dependence is reached for $N > 20$.

Although no attempt was made to obtain quantitative information concerning wake stability, certain observations of the unsteady force component were made. There were indications that relatively weak fields ($\sim .30$ web/m^2) were able to completely damp the dominant frequencies which had existed without an applied field. Similar results were obtained by Maxworthy (1962), who noted nonoscillatory wake behavior for $N \sim .5$. In addition, the suppression of turbulence due to an aligned field was noted by Globe (1961), who studied a turbulent pipe flow of mercury within a solenoid. On the other hand, we observed unsteadiness again for fields greater than $.50$ web/m^2 and the dominant frequency was seen to decrease monotonically as the magnetic field was further increased. The Strouhal number, S, of these strong field oscillations was quite small and dropped from a value of approximately .09 at $.50$ web/m^2 to .03 at $.70$ web/m^2.

These low frequency oscillations appeared when the steady drag and presumably the wake structure were considerably different from that with no field. It is possible that long wavelength disturbances were selectively amplified in the wakes, but detailed measurements of fluctuating velocity or magnetic field perturbation would be needed to understand this result. There was a definite trend toward lower values of S with increasing field, but no obvious dependence of S was found on N itself. Since C_D depended solely on N, for a given body, we concluded that the unsteady component was not a controlling factor in determining the steady drag.

We were able to find a simple drag law, $C_D \propto (N)^{\frac{1}{2}}$ independent of body shape, under the limiting conditions $N/Rm \gg 1$, $N/Re \ll 1$, and $N \gg 1$, $Re \gg 1$. This slightly viscous, strong field limit should be amenable to an analytical solution, but we have been unable to accomplish this. We shall, however, present a physical model which involves steady flow and contains the above result.

We can understand the fact that C_D was independent of body shape and the correlation with a simple solid body blockage correction if we consider the body to be surrounded by slender, relatively stagnant, constant pressure regions which are themselves separated from the outer flow by thin dissipation layers. Such a model was suggested in part by Childress, but required that the layer thickness, δ, be $\sim N^{-\frac{1}{2}}$ and $P \sim O(1)$, so that $C_D \sim O(1)$. Such a model is not unique and other scaling laws could apply which would agree with the experimental result that $C_D \propto (N)^{\frac{1}{2}}$.

Yonas (1966) has proposed that thicker layers permitting larger transverse velocities could exist, and that a consistent model explaining the measured C_D is given by $\delta \sim N^{-\frac{1}{4}}$ and $P \sim O(N^{\frac{1}{2}})$. The resulting equations can be solved to within two arbitrary functions and although a higher order expansion was suggested to remove the indeterminacy, this has yet to be done. Tamada (1962) has derived an expression for the behavior of the Bernoulli function, $H = P + V^2/2$. This becomes in the inviscid limit $\underline{V}.\nabla H = -N(\underline{V}x\underline{B})^2$. Since the Bernoulli function can only be reduced along a streamline by Joule loss, and since C_D increases as $N^{\frac{1}{2}}$ above 1.0, there must be a large suction on the downstream side of the body, the Bernoulli function changing from a large negative value within the downstream slug to the free-stream value across this layer. It is also conceivable that the largest part of the total dissipation in the flow would occur within these layers permitting one to calculate the drag from a simple energy balance. A solution incorporating the aspects of such a model, or an experimental verification using pressure and magnetic probes, is required.

Further verification of this "slug" behavior has been found in a separate experiment using a tank of sodium potassium eutectic within a uniform field.

For N as large as 600, a disk was moved within the tank toward the free surface and with the body roughly 10 diameters from it, an upwelling within the area subtended by the body was noted. A downwelling of the same size was observed with the disk moving away from the surface. This demonstrated the usefulness of a tow tank facility for the study of N flows, the existence of weakly damped Alfvén waves and of a concentrated forward wake.

4. Conclusion

A measurement of the drag coefficient of spheres and disks over a wide range of N has been carried out. We found a simple drag law, $C_D \propto N^{\frac{1}{2}}$ independent of body shape, corresponding to the limit $N/Rm >> 1$, $N/Re << 1$, $N >> 1$, $Re >> 1$. A physical model explaining such a result was presented which involved slender, relatively stagnant regions which increase in length as N is increased, and preliminary observations of this phenomenon were described.

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FIGURE CAPTIONS

Fig. 1 Drag balance test section.

Fig. 2 Drag coefficient vs interaction parameter for spheres.

Fig. 3 Sphere drag coefficient corrected for blockage vs interaction parameter.

	$\frac{d}{D}$
○	0.125
△	0.250
□	0.375

Fig. 4 Disk drag coefficient vs interaction parameter.

	$(N\ Re)^{\frac{1}{2}}$
×	0
△	297
○	505
○	789
□	1127

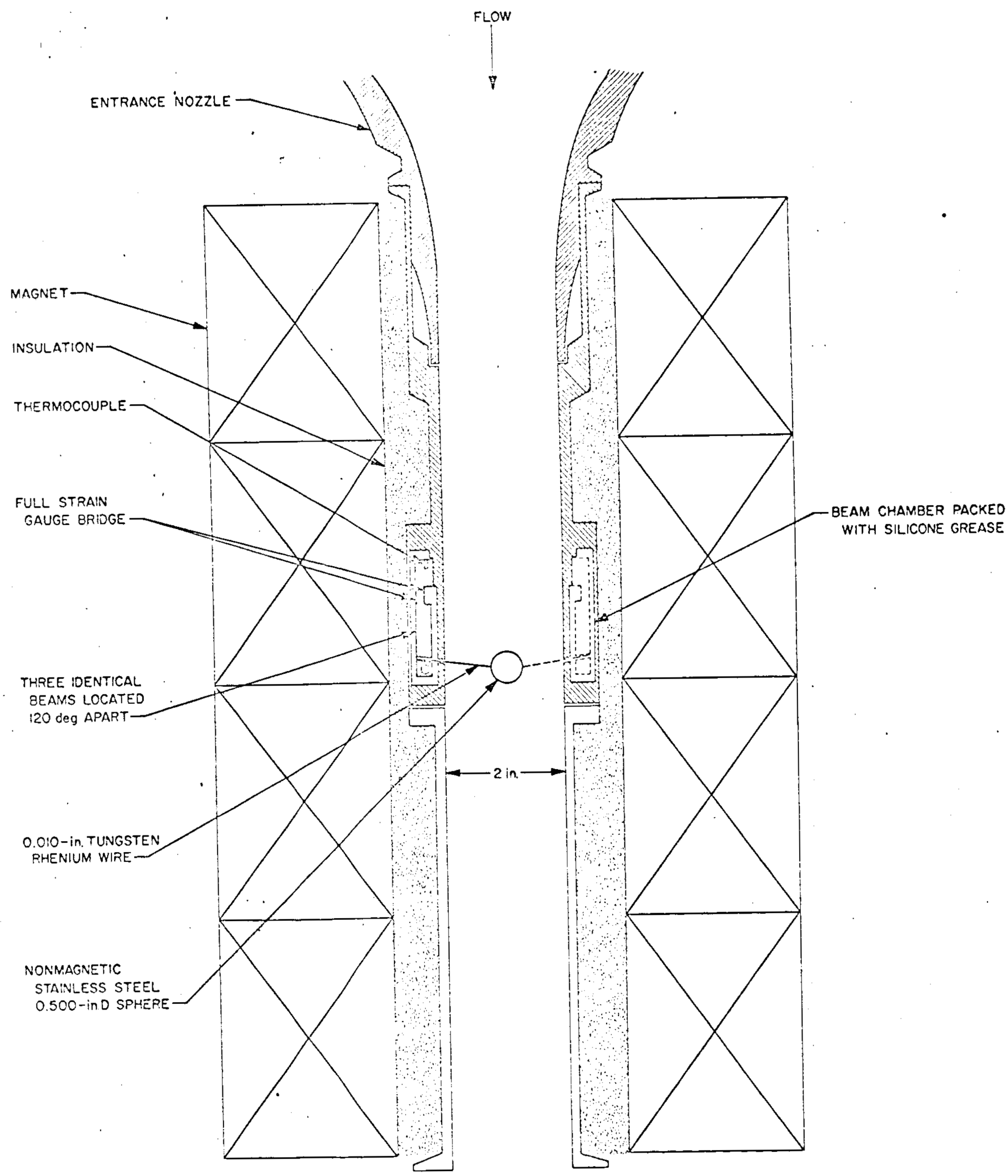
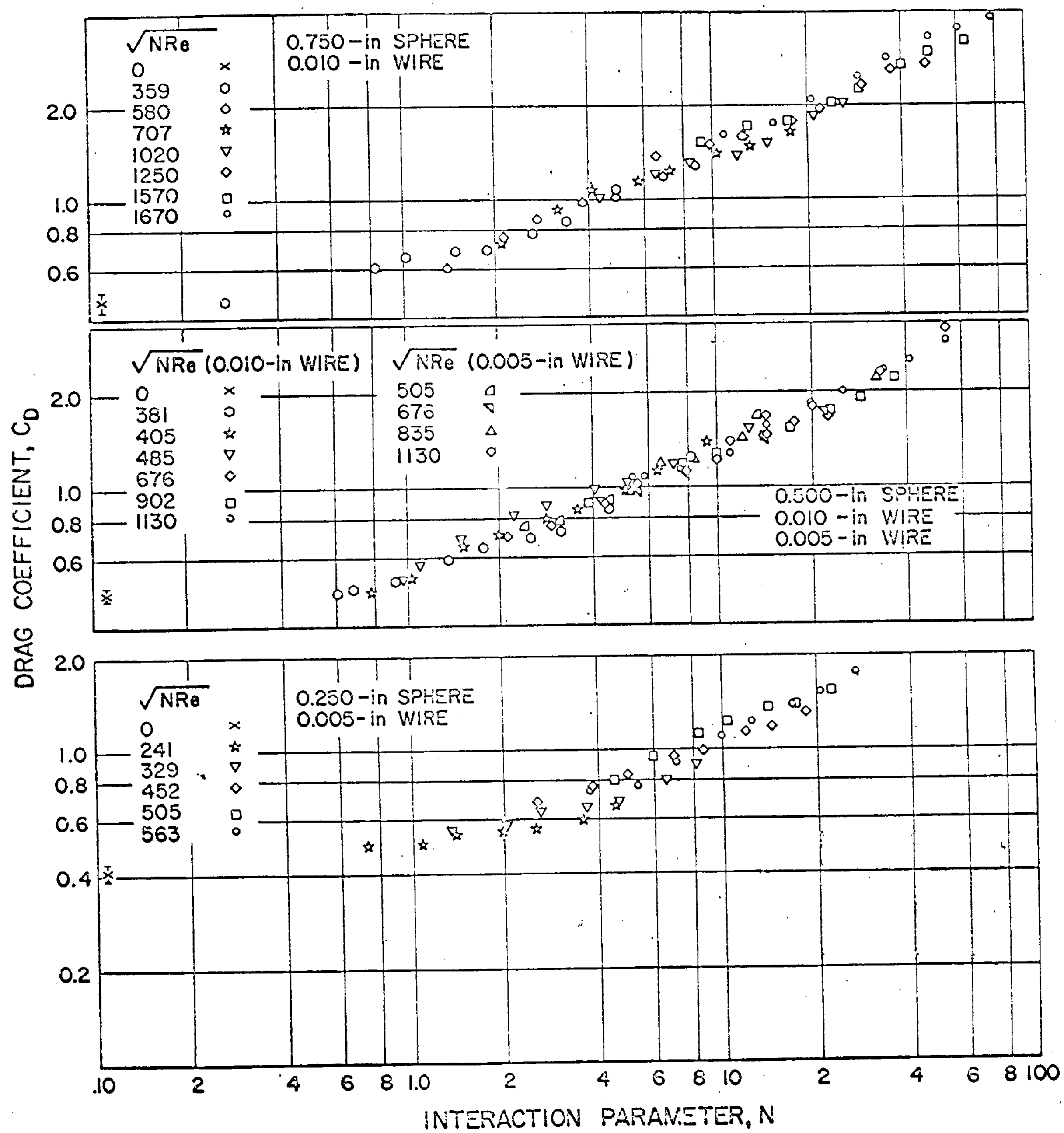

FLOW
ENTRANCE NOZZLE
MAGNET
INSULATION
THERMOCOUPLE
FULL STRAIN
GAUGE BRIDGE
THREE IDENTICAL
BEAMS LOCATED
120 deg APART
0.010-in. TUNGSTEN
RHENIUM WIRE
NONMAGNETIC
STAINLESS STEEL
0.500-in D SPHERE
BEAM CHAMBER PACKED
WITH SILICONE GREASE
2 in.

Fig. 1



√NRe
0 ×
359 ○
580 ◇
707 ☆
1020 ▽
1250 ◇
1570 □
1670 ○
0.750-in SPHERE
0.010 -in WIRE
2.0
1.0
0.8
0.6
√NRe (0.010-in WIRE)
0 ×
381 ○
405 ☆
485 ▽
676 ◇
902 □
1130 ○
√NRe (0.005-in WIRE)
505
676
835
1130
0.500-in SPHERE
0.010 -in WIRE
0.005 -in WIRE
√NRe
0 ×
241 ☆
329 ▽
452 ◇
505 □
563 ○
0.250-in SPHERE
0.005-in WIRE
0.4
0.2
DRAG COEFFICIENT, C_D
.10 2 4 6 8 1.0 2 4 6 8 10 2 4 6 8 100
INTERACTION PARAMETER, N

Fig. 2

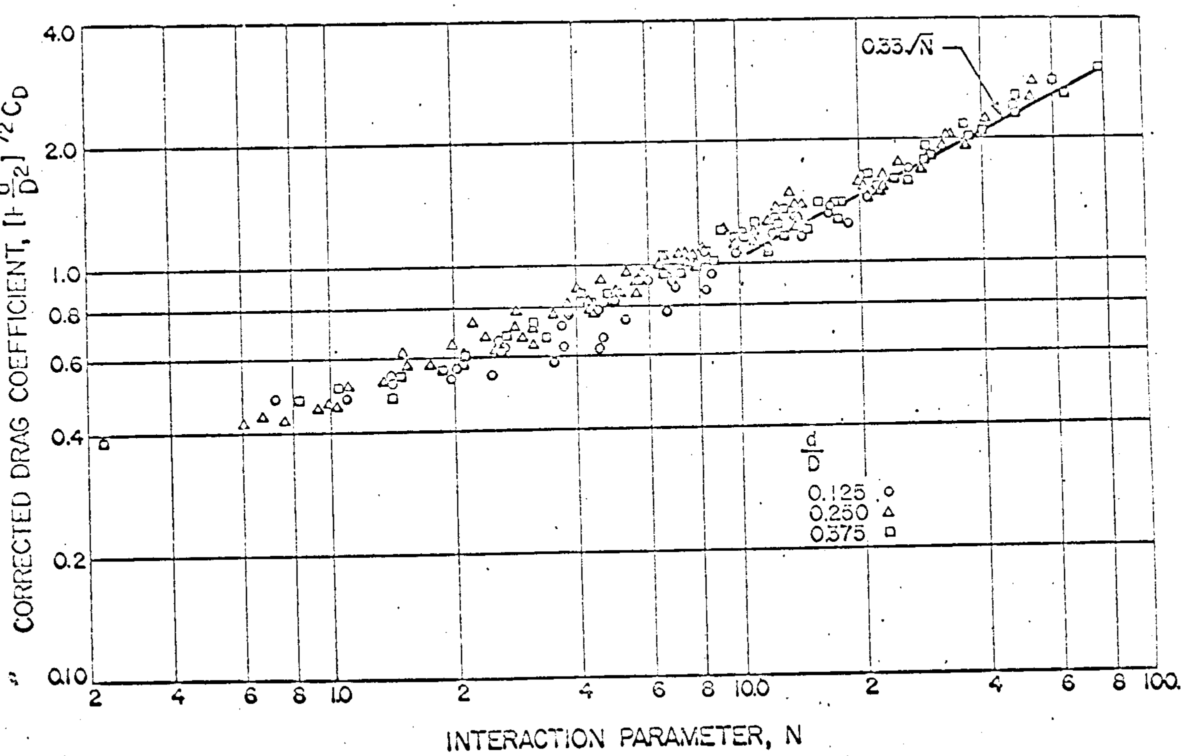
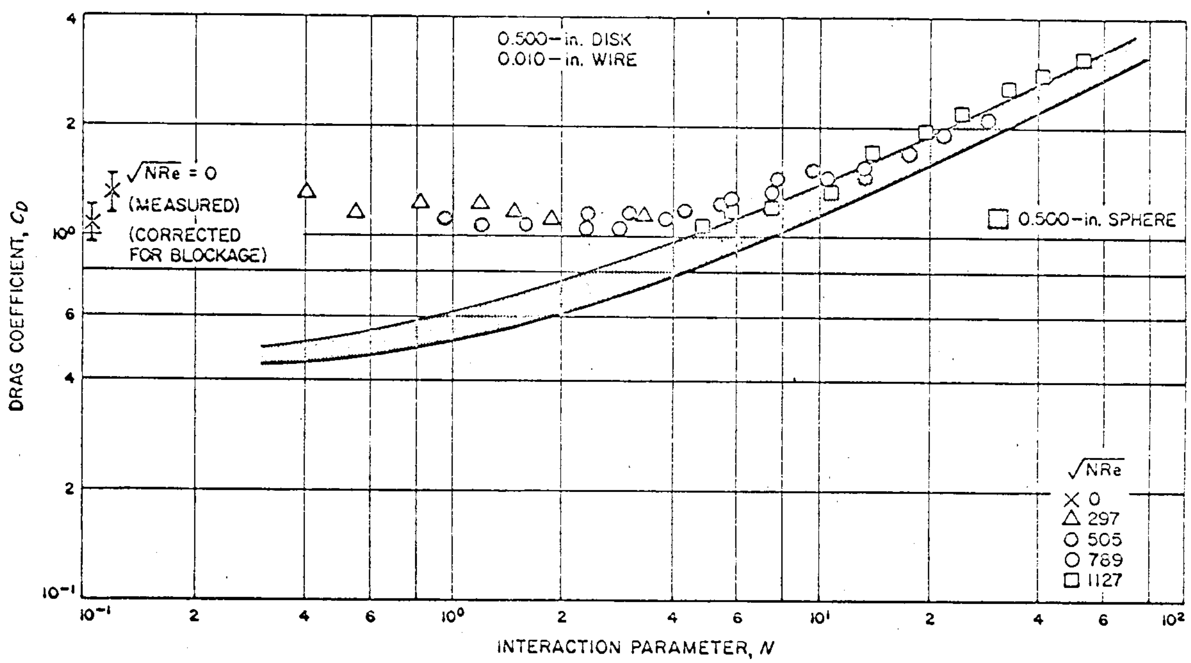

4.0
2.0
1.0
0.8
0.6
0.4
0.2
0.10
CORRECTED DRAG COEFFICIENT, $[1-\frac{d^2}{D^2}]^{3/2} C_D$
2
4
6
8
1.0
2
4
6
8
10.0
2
4
6
8
100.
INTERACTION PARAMETER, N
$0.33\sqrt{N}$
$\frac{d}{D}$
0.125 ○
0.250 △
0.375 □

Fig. 3



0.500-in. DISK
0.010-in. WIRE
$\sqrt{N_{Re}}$ = 0
(MEASURED)
(CORRECTED FOR BLOCKAGE)
0.500-in. SPHERE
$\sqrt{N_{Re}}$
X 0
△ 297
O 505
O 789
□ 1127
DRAG COEFFICIENT, C_D
INTERACTION PARAMETER, N

Fig. 4